Study of strength and its reliability of SiC fiber bundle by experimental and Monte-Carlo simulation approach

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A B S T R A C T
Tensile tests and Monte-Carlo simulation on the strength and its reliability of the SiCVD (formed by chemical vapor deposition) fiber bundle were performed. The experimental and simulation results were compared and analyzed by fiber bundle strength theories. The experimental results showed that the strength of the SiCVD fiber bundle was decreased as the number of fibers in the bundle was increased; while in the process, Weibull shape parameter of the bundle strength was increased. The experimental results were in good agreement with the Monte-Carlo simulation. In addition, Monte-Carlo simulation was used to clarify the detailed relationship between the strength of the SiCVD bundle and the number of fibers in the bundle and the simulation results were compared with that of Coleman theory. The comparison revealed that the strength of the SiCVD bundle converged to the value of Coleman theory as the number of fibers in the bundle increased; at the same time, the rate of convergence was increased as Weibull shape parameter was increased. Furthermore, the relationship between the strength of the SiCVD fiber bundle and Weibull shape parameter of the SiCVD fiber strength was also examined. It was found that the strength of the SiCVD fiber bundle increased as Weibull shape parameter was increased although the number of fibers in the bundle was countable. Finally, the breaking-down process and the number of broken-fibers in the bundle depended on the number of fibers the bundle and Weibull shape parameter of the SiCVD fiber strength.

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1. Introduction

In recent years, fiber-reinforced composites are increasingly applied in the industries of aerospace and aviation for their excellent mechanical properties including high specific stiffness, high specific strength and so on. However, the properties of fibers adopted in fiber-reinforced composites are distinct from that of the matrix and the strength of fibers shows divergence. Therefore, it is very important to evaluate strength and its reliability of fibers and fiber bundles when mechanical properties of fiber-reinforced composites are discussed. In evaluation of fiber strength distribution, Weibull distribution [1] is one of the most frequently used theories. After that, Daniel [2] developed his classic theory on fiber bundle strength which greatly improved the simple Weibull theory. Daniels’ theory proved that the bundle strength was asymptotically distributed with a normal distribution. Thereafter, Coleman [3] established his theory on the basis of Weibull theory and Daniels’ theory and represented the relationship between the strength of fiber bundle and strength of the individual fibers as indicated in Eq. (1).

\[
\frac{\sigma_b}{\sigma_f} = \left( \frac{1}{m_f} \right)^{1/m_f} \frac{1}{\Gamma(1+1/m_f)}
\]  

(1)

where \(\sigma_b\) and \(\sigma_f\) are bundle strength and strength of fiber in the bundle, \(m_f\) is Weibull shape parameter of fiber strength, \(e\) is the base of the natural logarithms, and \(\Gamma\) is gamma function. From Eq. (1), it can be known that the ratio of fiber bundle strength and strength of fiber in the bundle only depends on Weibull shape parameter \(m_f\). Since then, Coleman’s theory has been often used to discuss fiber bundle strength as the base of mechanical models of fiber-reinforced composite [4–8]. In the last few years, many researchers have made great progresses in the study of strength and its reliability of fiber-reinforced composites, for example, Hwang [9] studied size effect on fiber strength of composite; Porwal [10] applied computer simulation method to analyze the statistical strength of fiber bundles; the attention of Vorechovsky [11] is focus on small failure probabilities and the related probabilistic distributions of the strength of composites and presented an extension of Weibull
theory by introducing into a statistical length scale; Dassios [12]
investigated the strength of Al₂O₃ fiber bundle at room and elevated temperatures.

However, Daniels’ theory and Coleman’s theory were based on the assumption that the number of fibers in the bundle is numerous. While, till now, any report on the exact number of fibers in the bundle has not appeared.

In the study, a series of tensile tests and Monte-Carlo simulations were carried out for SiCCVD fiber bundle. Bundle strength and its reliability were examined and discussed. Besides the influence of the number of fibers in the bundle on the strength and its reliability of fiber bundle were also investigated. Finally, the results of experimental and Monte-Carlo simulation were compared with Coleman’s theory.

### 2. Experimental and Monte-Carlo simulation approach

#### 2.1. Tensile test of SiCCVD fiber bundle

SiCCVD fiber, produced by Textron Specialty Materials (type: SCS-9, diameter: 70 μm), was used as the fiber in the studied SiC bundle. A series of tensile tests were carried out at room temperature in a screw-driven constant crosshead tensile testing machine (Shimadzu, AG-5000ES) with a constant tensile speed of 0.5 mm/min and a gauge length of 30 mm for the fibers in the bundles which included 13, 50, 100 and 200 fibers, respectively. Fig. 1 shows tensile specimens and their loading modes of the SiCCVD fiber and the bundle. From Fig. 1(a), it can be seen that Al tabs with a length of 30 mm were adhered to both of the tensile specimens’ ends in the case of individual fiber; as for the bundle, fibers were arranged in parallel along the tensile direction and formed a bundle of fibers. To avoid any damage to the individual fiber or the fiber bundle before tensile tests, supporting paperboards were used to one side of the tensile specimens and were burnt just before tensile tests began.

All the studied fibers were selected at random from numerous numbers of SiCCVD filaments which were cut into ones with a length of 70 mm. Besides, the number of the tensile specimens for each group of fiber bundle was over 40.

#### 2.2. Monte-Carlo simulation approach

A series of Monte-Carlo simulation was carried out for tensile tests of the SiCCVD fiber bundles. In the simulation, it was assumed that the fiber strength complied with two-parameter Weibull distribution shown as Eq. (2). Therefore, the strength of the individual fiber can be obtained in computer program from the inverse function of Weibull distribution shown as Eq. (3).

$$F(\sigma_f) = 1 - \exp \left\{ - \left( \frac{\sigma_f}{\sigma_0} \right)^{m_f} \right\}$$  \hspace{1cm} (2)

$$\sigma_f = \sigma_0 \left\{ \ln \left( \frac{1}{1-Z} \right) \right\}^{1/m_f}$$  \hspace{1cm} (3)

where $$\sigma_f$$ is strength of fiber, $$m_f$$ and $$\sigma_0$$ are Weibull shape parameter and scale parameter of fiber strength, respectively. Z is uniform random number in (0, 1). It was assumed that the diameter and gauge length of the fiber were constant. Also, it was presumed that the given load applied to each of the fiber in the bundle uniformly and the load applying to the breaking fiber would redistribute amongst the surviving fibers. The parameters obtained from the tensile tests of individual SiCCVD fiber were used in the present simulation as listed in Table 1. In the simulation, Weibull scale parameter, $$\sigma_0$$ was a constant and substituted by the experimental value, 4.455 GPa. Based on the experimental values, Weibull shape parameter, $$m_f$$ was given values of 1, 3, 6, 9.095 and 15, respectively.

Fig. 2 shows the flow chart of the present Monte-Carlo simulation. Where, $$\sigma$$, $$\varepsilon$$, E, P and A are mean strength, strain, Young’s modulus, load and sectional area, respectively. Also, subscript f, s,
b, fu and max mean fiber, slack, bundle, fracture and maximum, respectively. For example, $\sigma_f$ expresses the strength of the $i$th fiber in a fiber bundle, and is obtained from Eq. (3).

The procedure of the simulation is shown as follows. Firstly, strength of each fiber is given after inputting the parameters of the SiCVD fiber. Secondly, to simulate tensile test of the fiber bundles, which have slack for individual fiber and the slack is added to the fracture strain for individual fiber comparing with experimental results. The slacks are given by multiplying $r \times \varepsilon_{fu}$ to uniform random number $Z$ in $(0, 1)$, $r$ is the constant. Where, $\varepsilon_{fu}$ is the ultimate fracture strain of the SiCVD fiber and equals to the ultimate strength divided by elastic modulus of the fiber, $\sigma_{fu}/E_f$. $r\varepsilon_{fu}$ represents the level of the slacks, and was substituted by 0.55 in the case of slacks or by zero in the case of no slacks according to pre-simulation. Thereafter, the order statistic of fiber breaking strains including the slacks of the fibers in the bundle is given. Then the displacement increment is given to the fiber bundle at the step by following the order statistic of all the fiber-breaking including the slacks. At the same time, the load increment is given. After a fiber breaks, the load acting on the breaking fiber will redistribute to the surviving fibers which have a smaller slack strain than the bundle strain at that step. Stress and strain of the fiber bundle can be calculated by the accumulation of displacement increment and load increment at each step, respectively. The strength of fiber bundle defined as the maximum load divided by sectional area of the fiber bundle. Monte-Carlo simulations were carried out for 50 times repeatedly.

Relationship between the fracture process of the SiCVD fiber bundle and the fibers in the bundles without or with slacks is shown as Fig. 3. From the figures, it can be seen that slacks had strongly effect on the fracture process, the ultimate strength of the fiber bundle with slacks was lower than that without slacks. The result was in agreement with the study of Phoenix [6]. The tensile tests in the study were more close to the situation of Fig. 3(b). That proved that slacks exactly existed in the practical tensile tests. In the following discussion, slacks were also considered to make the simulation and the experiment agreed with each other.

3. Results and discussion

Fig. 4 shows stress–strain curves of the SiCVD fiber bundles in tensile tests. Though the bundle strain was calculated by using displacement of the crosshead during tensile tests, it can be seen clearly that the slope of curves became moderate as the number of fibers in the bundles increased. It hints that the slacks of fibers in the bundles went up as the number of fibers in the bundles increased. Also, before the maximum stress of the bundles appeared, there were abrupt drops on the curves due to the fracture of fibers in the bundles. Relationship between the bundle strength and the number of fibers in the bundles in experiment and simulation is shown in Fig. 5. From this figure, it can be seen that the bundle strength decreased with the increase of the number of fibers in the bundles, but the descent rate became lower when the number of fibers in the bundles was over 100. Although the bundle strength obtained from the simulation was larger than the experimental ones in the case of no slacks, they were well consistent with each other by introducing the slacks. The result that bundle strength decreased as the number of fibers in the bundle increased was also consistent with the correction described by Phoenix [13]. Also, the relationship between Weibull shape parameter of bundle strength and the number of fibers in the bundles is shown in Fig. 6. Weibull shape parameter of bundle strength increased with the increase of the number of fibers in the bundles for both of experiments and Monte-Carlo simulation. After introducing slacks, the simulation results were more close to the experimental ones.

![Fig. 3. Relationships between the fracture process of the SiCVD fiber bundle and the fibers in the bundle without or with slacks: (a) without the slacks and (b) with the slacks. Denotation $\varepsilon, \sigma, b, f$ and $u$ represent strain, stress, bundle, fiber and ultimate, respectively.](image1)

![Fig. 4. Stress–strain curves of the SiCVD fiber bundles in tensile tests.](image2)

![Fig. 5. Relationship between the bundle strength and the number of fibers in the bundles, $r$ represents level of slacks and $m_f$ is Weibull shape parameter of the SiCVD fiber strength.](image3)
From the results above, it can be concluded that slacks existed in the fiber bundles of the experiments. It also proved that the present Monte-Carlo approach was applicable to simulate the tensile tests of the SiCCVD fiber bundles. However, the simulation was only carried out for no slacks below in order to compare with the results of Coleman's theory because there was no the concept of slacks in Coleman's theory.

To compare with Coleman’s theory, the relationship between the strength of the bundle and the number of fibers in the bundles was examined by the simulation by changing Weibull shape parameter of the fiber strength. The results are showed in Fig. 7. Bundle strength was denoted as the ratio of bundle strength from the simulation to that from Coleman’s theory. It can be noted that bundle strength decreased and converged to Coleman’s theory as the number of fibers in the bundles increased. Besides, bundle strength converged to Coleman’s theory with a higher rate when Weibull shape parameter was larger.

Fig. 8 shows the relationship between the bundle strength and Weibull shape parameter of fiber strength. Here, bundle strength is denoted as the ratio of bundle strength ($\sigma_b$) and fiber strength ($\sigma_f$). From the figure, it can be seen that bundle strength went up as Weibull shape parameter increased even though the number of fibers in the bundles was countable; besides, the curve of bundle strength was more closed to that of Coleman’s theory with larger number of fibers in the bundles.

According to the assumption and method of Daniels [2] and Coleman [3], coefficient of variation (C.V.) of bundle strength is given as Eq. (4).

$$C.V. = \left[ \exp \left( \frac{1}{m_f} \right) - 1 \right]^{1/2} N_f^{-1/2}$$  \hspace{1cm} (4)

where $N_f$ is the number of fibers in the bundle. From this equation, it can be understood that the divergence of bundle strength is only related to Weibull shape parameter of the fiber strength and the number of fibers in the bundle. From the equation, it can be concluded that, on one hand, C.V. will decrease as the number of fibers in the bundle increases when $m_f$ is constant; on the other hand, when the number of fibers in the bundle is constant, C.V. will also decrease with the increase of $m_f$. Therefore, it can be concluded from Eq. (4) and Fig. 6 that C.V. of the fiber bundle was decreased as the number of fibers in the bundle was increased.

Coefficient of variation of bundle strength from Monte-Carlo simulation and the calculated results from Eq. (4) is shown in Fig. 9.
Coefficient of variation of bundle strength went down with the increase of the number of fibers in the bundles and the results from the simulation and the calculation are relatively similar with each other.

To discuss the relationship between the fracture process, the number of broken-fibers in the bundles against the number of fibers in the bundles, fracture stress of fiber was recorded in the order number of broken-fiber in each simulation. Then average fiber stress and Weibull shape parameter were calculated by Weibull distribution. Relationships between the average fiber stress, Weibull shape parameter of broken-fiber stress against the order number of broken-fibers are shown in Figs. 10 and 11, respectively. From Fig. 10, it can be seen that the average fiber stress increased as the order number of broken-fibers, that means fracture stress of the bundle would be increased as the number of broken-fibers increased. From Fig. 11, Weibull shape parameter of broken-fiber stress was found to increase as the number of broken-fibers increased. It hints that the divergence of fiber bundle strength was decreased. Based on the results above, it was obvious that bundle strength depended on the number of fibers in the bundles more increasingly with the increasing of the number of broken-fibers in the bundles. In fracture process, strength of fiber bundle became higher and its divergence became lower.

Fig. 12 shows the relationship between the number of fibers in the bundles and the number of broken-fibers till maximum stress appeared. It can be found that the number of broken-fibers in the bundle went up as the number of fibers in the bundles was increased or with the decrease of Weibull shape parameter of the SiCCVD fiber strength. From the results above, it can be concluded that the breaking-down process of the SiCCVD fiber bundle and the number of broken-fibers in the bundles before maximum stress appeared depended on the number of fibers in the bundles and Weibull shape parameter, $m_f$ of the SiCCVD fiber strength.

4. Conclusions

Based on the experiment and Monte-Carlo simulation above, some important results can be concluded as follows.

1. In the process of the number of fibers in the bundles was increased, the strength of the SiCCVD fiber bundle decreased and Weibull shape parameter of bundle strength increased.

2. The strength of the SiCCVD fiber bundle converged to Coleman theory as the number of fibers in the bundles was increased and the rate of convergence was higher when Weibull shape parameter of the SiCCVD fiber strength had a larger value.

3. The strength of the SiCCVD fiber bundle was increased with the increase of Weibull shape parameter of the SiCCVD fiber strength as Coleman theory's description even when the number of fibers in the bundles was countable.

4. The fracture process and the number of broken-fibers in the bundle before maximum stress appeared depended on the number of fibers in the bundles and Weibull shape parameter, $m_f$ of the SiCCVD fiber strength.

References